## Master project: A topological analysis of the El Niño Southern Oscillation

**Summary of project**
The El Niño Southern Oscillation (ENSO) is one of the most important and studied phenomena of atmospheric variability, as it influences variations in temperature at a global scale. Developing a better understanding of ENSO is crucial in being able to better model the climate's response to continued anthropogenic emissions. In this project we propose to use techniques from the field of Topological Data Analysis to gain better understanding in existing models of ENSO. The project will be supervised by Hannah Christensen, Associate Professor in the Physics Department at the University of Oxford, and Nina Otter, Inria Starting Faculty, DataShape, Inria-Saclay.

**Mentors:**

- Hannah Christensen, Department of Physics, University of Oxford
- Nina Otter, DataShape, Inria-Saclay, and Laboratoire de Mathématiques d'Orsay

**Objectives of internship :**

- Use PH-based methods to analyse bifurcation plots of ENSO models, both in the deterministic as well as stochastic setting.
- Computation and comparison of PH with respect to sublevel set filtrations of the ENSO time series arising from observational data and time series arising from ENSO models.
- Participation and brief progress-report talk at annual DataShape seminar, May 2025, Porquerolles.
- Redaction of a research paper, to be submitted to a climate science research journal.

**Methods:**

- Computation of persistent homology for sublevel set filtrations of time series
- Computation of Wasserstein and bottleneck distances between diagrams
- Cluster analysis
- Crocker plot analysis

**Required skills:**

- Proficiency in Python
- Proficiency in MATLAB desired, but not required
- Strong knowledge in algebraic topology or in climate modelling

**Timeline** (max duration: 5 months)
**Start:** 01.04.2025
**End:** 31.08.2025

**Funding source:** Programme de recherche Mathématiques en interaction, PEPR Maths-Vives (ANR)

**Location of internship (host institution):** Laboratoire de Mathématiques d'Orsay
Address : rue Michel Magat 307, Orsay, France

**Detailed description**

Topological Data Analysis (TDA) is a field that uses insights from topology — the mathematical area that studies abstract shapes — to develop representations of data that are computable and robust in an appropriate sense [1]. Persistent homology (PH) is, arguably, one of the most successful methods used in Topological Data Analysis, and it is being increasingly applied to a variety of data analysis problems. See the DONUT database [2] for a vast collection or real-world applications of PH, and [3] for an overview of the main algorithms, software implementations and computational problems studied in PH.

In the current modelling of the climate, there are many challenges that are due to the size and the noise in the data. For many outcomes of interest (for instance, studying how weather regimes will change in the Euro-Atlantic sector) there is a large amount of noise in the system (so-called *internal variability)* which is often on the same order of magnitude as the signal, making it challenging to detect robust signals. This is a well-known problem in the field that leads to studies often not drawing similar conclusions, and creates a gap between the theoretical modelling of the climate and how well this can be used in practice, for instance for impact studies or predictions. Techniques from TDA are particularly well-suited to studying climate data, because they provide efficient and computable representations of data that are robust in an appropriate sense to noise. Such techniques have already been used to successfully individuated weather regimes [4, 5]. For an overview of how techniques in TDA have been used for modelling the climate, we refer to the survey [6].

The El Niño Southern Oscillation (ENSO) is a set of coupled ocean-atmosphere phenomena characterised by an irregular cycle of warming (El Niño) and cooling (La Niña) in the eastern tropical Pacific together with a corresponding variation in sea level pressure. ENSO significantly impacts global weather patterns and it is one of the main phenomena of atmospheric variability studied and modelled by climate scientists. A better understanding of the ENSO dynamics is crucial in the modelling of the climate's response to continued anthropogenic emissions.

The goal of this project is to use techniques from TDA, and in particular PH, to study models of the El Niño Southern Oscillation, and how they compare to ENSO data. The models are delayed-oscillator models; in particular, they depend on a parameter that encodes the strength of the coupling between the ocean and the atmosphere [7]. We foresee that during the project the student will use PH-based methods (Crocker plots [8], clustering based on Wasserstein or bottleneck distances between persistence diagrams) to analyse bifurcation plots of ENSO models, both in the deterministic as well as stochastic setting. A further analysis that will be performed is the computation and comparison of PH with respect to sublevel set filtrations of the ENSO time series arising from observational data and time series arising from ENSO models.

## References


[1] Gunnar Carlsson. Topology and data. Bulletin of the American Mathematical Society, **46**(2):255–308, 2009

[2] Barbara Giunti, Janis Lazovskis, and Bastian Rieck. DONUT: Database of Original & Non-Theoretical Uses of Topology, 2022. https://donut.topology.rocks

[3] Nina Otter, Mason A. Porter, Ulrike Tillmann, *et al.* A roadmap for the computation of persistent homology. *EPJ Data Sci.* **6**, 17, 2017

[4] Kristian Strommen, Matthew Chantry, Joshua Dorrington, and Nina Otter. A topological perspective on weather regimes. *Climate Dynamics*, pages 1–31, 2022.

[5] Davide Faranda, Thép Lacombe, Nina Otter, Kristian Strommen, Climate science at the interface between dynamical systems theory and topological data analysis, *AMS Notices*, **71**:2,2024

[6] Michael Ghil, Denisse Sciammarella, Review article: Dynamical systems, algebraic topology and the climate sciences, *Nonlinear Processes in Geophysics*, **30:**399–434, 2023

[7] Hannah Christensen, Judith Berner, Danielle Coleman, Tim Palmer, Stochastic Parameterization and El Niño–Southern Oscillation, *Journal of Climate,* 2017

[8] Ismail Güzel, Elizabeth Munch, Firas Khasawneh, Detecting bifurcations in dynamical systems with crocker plots. *Chaos: An Interdisciplinary Journal of Nonlinear Science,* **32**(9), 2022